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# Integration of APOS, RME, and Digital Learning: A Strategic Model to Enhance Computational Thinking

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**ABSTRACT**

Computational Thinking (CT) is essential for 21st-century problem-solving but remains underdeveloped in abstract mathematics courses like linear algebra. This study addresses the gap by integrating APOS theory, Realistic Mathematics Education (RME), and digital tools into a strategic learning model to enhance CT and conceptual understanding. A Design-Based Research (DBR) approach was employed to develop, implement, and evaluate the model with 73 first-year university students enrolled in a linear algebra course at Institut Bakti Nusantara. Participants were grouped by class section into experimental (APOS–RME–digital integration) and comparison (conventional digital instruction) groups. Data were collected through pre-post CT assessments, classroom observations, surveys, and learning analytics. Students in the experimental group showed significantly higher gains across five CT components—abstraction, decomposition, algorithmic design, evaluation, and generalization—compared to the control group, with medium to large effect sizes. Improved conceptual understanding of vectors and linear transformations was also observed. Learners reported high usability and perceived instructional value, particularly in contextual and interactive tasks. Implementation challenges related to digital access and skills were mitigated through structured onboarding and offline resources. The findings demonstrate that a well-integrated APOS–RME digital model can systematically develop CT and mathematical understanding. High engagement and usability support its practical viability in higher education settings. This model offers a scalable, theory-informed framework for digital mathematics instruction. Future research should explore long-term impacts, equity strategies, and cross-institutional adoption to further enhance its applicability and sustainability.



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## 1. INTRODUCTION

The rapid transformation of education in the digital era has introduced both new opportunities and challenges in preparing students with essential 21st-century competencies. These competencies include critical thinking, problem solving, collaboration, communication, and digital literacy—skills that are increasingly demanded across disciplines and professions (Tariq et al., 2024). In particular, *computational thinking* (CT) has emerged as a foundational cognitive skill, closely tied to the ability to formulate, analyze, and solve problems using logical and algorithmic approaches. As defined by Shute et al. (2021), CT encompasses five core components: abstraction, decomposition, algorithmic design, evaluation, and generalization. These components are vital not only in computer science but also in mathematics, engineering, and other STEM fields where students must model complex problems and reason through multi-step processes (Weintrop et al., 2021).

Despite its importance, the development of CT in mathematics education—particularly in higher education—remains uneven. Many undergraduate students struggle to cultivate CT skills in abstract mathematics courses such as linear algebra, especially in topics like vectors and linear transformations. This difficulty is often rooted in the predominance of procedural, teacher-centered pedagogies, which emphasize symbolic manipulation over conceptual understanding or problem-based reasoning (Lu et al., 2023; Tuktamyshov & Gorskaya, 2023). Without access to tasks that require deep modeling, iterative reasoning, or contextual application, students tend to engage with mathematical concepts in isolation, hindering their ability to transfer knowledge to authentic, real-world situations.

While digital technologies such as Learning Management Systems (LMS), GeoGebra, and MATLAB are increasingly embedded in university mathematics courses, their instructional use often remains limited to content delivery, assignment submission, or passive visualization, rather than supporting active, exploratory, and computational forms of learning (de Jong & Jeuring, 2020). As a result, opportunities to integrate digital tools in ways that systematically foster CT and deepen conceptual learning are frequently underutilized.

To address this challenge, two well-established theoretical frameworks in mathematics education offer promising avenues: APOS theory (Action, Process, Object, Schema) and Realistic Mathematics Education (RME). APOS theory provides a cognitive-developmental model for understanding how students internalize and restructure mathematical knowledge, moving through sequential stages of action, process, object formation, and schema integration (Dubinsky & McDonald, 2001). On the other hand, RME emphasizes the importance of starting with real-life, meaningful problems to support the process of *mathematization*—bridging everyday contexts with formal mathematical representations (Gravemeijer & Doorman, 1999; van den Heuvel-Panhuizen & Drijvers, 2022). Together, these theories have shown promise in enhancing conceptual understanding and problem-solving skills, yet research exploring their combined implementation within digital learning ecosystems remains scarce (van Zanten & van den Heuvel-Panhuizen, 2021).

Initial needs analysis conducted with 73 first-year university students at Institut Bakti Nusantara confirmed widespread difficulties across all five CT components. Students particularly struggled with abstracting vector operations into algorithmic procedures and applying them in real-world contexts such as GPS mapping or drone navigation. Additionally, challenges emerged in decomposing multidimensional mathematical problems and constructing algorithmic solutions using tools like MATLAB. These findings echo international research that highlights a persistent gap in students' ability to link formal mathematical content with practical or computational reasoning (Lu et al., 2023).

Previous efforts to address CT in education have often focused on isolated interventions, such as teaching programming languages, using standalone applets, or implementing short-term project-based tasks (Tariq et al., 2024). While valuable, these approaches typically lack a unifying pedagogical structure and do not fully leverage the synergies among cognitive theory, contextual learning, and digital tool integration. Recent bibliometric studies also indicate a lack of robust linkage between CT and foundational learning theories like APOS and RME in current mathematics education literature (van Zanten & van den Heuvel-Panhuizen, 2021), suggesting an urgent need for a more comprehensive, theory-informed instructional strategy.

In response to this gap, the present study proposes and evaluates a strategic digital learning model that explicitly integrates APOS theory, RME, and commonly available digital tools—including LMS, GeoGebra, and MATLAB—to enhance CT in abstract mathematics learning. The goal of this model is not only to improve CT outcomes but also to deepen conceptual understanding and facilitate engagement with real-world mathematical contexts, supporting the development of transferable, future-ready skills. The integration is grounded in a Design-Based Research (DBR) methodology that allows for iterative refinement and alignment of instructional design with empirical insights (McKenney & Reeves, 2019; Easterday et al., 2021).

To guide this study, the following research questions were formulated:

1. Does the integration of APOS, RME, and digital learning significantly enhance CT components compared to traditional instruction?
2. How does the integrated model affect students' conceptual understanding of abstract mathematical topics?
3. What are students' perceptions regarding the feasibility and usefulness of the integrated model in mathematics learning?

Through this inquiry, the study aims to contribute a replicable, scalable framework for theory-informed digital instruction in mathematics education and to provide evidence-based insights for educators, curriculum designers, and policymakers committed to advancing CT and conceptual learning in the digital age.

## 2. METHODS

### *2.1 Research Design and General Background*

This study employed a Design-Based Research (DBR) methodology to develop and evaluate a digital learning model integrating APOS theory, Realistic Mathematics Education (RME), and digital platforms to enhance Computational Thinking (CT) among higher education students. The DBR method was selected for its systematic cycles of design, implementation, evaluation, and refinement, thus balancing theory-informed innovation with practical application (McKenney & Reeves, 2019; Easterday et al., 2021). The research progressed through four key stages: (1) preliminary analysis (needs assessment and literature review); (2) design and development of an integrated model using LMS, GeoGebra, and MATLAB; (3) implementation via pilot testing; and (4) evaluation, including expert review, empirical tests, and iterative refinement. Instruction focused on vectors and linear transformations—topics known for student difficulty in both conceptual and procedural features (Lu et al., 2023; Tuktamyshov & Gorskaya, 2023). Employing a mixed-methods design, the study combined quantitative (CT skill gains) and qualitative (learning experiences) data to provide comprehensive evidence regarding the model's effectiveness (Alam et al., 2025; Tariq et al., 2024).

### *2.2 Participants*

Participants were 73 first-year undergraduates enrolled in Linear Algebra (vectors and linear transformations) at Institut Bakti Nusantara, semester 2024/2025. A census approach included all students meeting these inclusion criteria: (a) active enrolment, (b) first-year status, and (c) consent to complete all assessments. Exclusion criteria were incomplete attendance or missing data. For the DBR evaluation cycle, intact class sections—not individuals—were cluster-assigned to experimental or comparison groups to minimize contamination and logistical disruption, as students in the same class naturally share schedules, assignments, and exposure to instruction (Dreyhaupt et al., 2017; Wikman et al., 2025; Bishop, 2023). Full individual randomization was infeasible due to university scheduling constraints, preserving authentic classroom structure and facilitating valid, pragmatic comparisons.

Sections were assigned (at the class level) to either the APOS–RME digital model or business-as-usual digital instruction; the latter followed the standard LMS-based curriculum and did not include explicit APOS or RME elements. Baseline academic indicators confirmed initial equivalence between groups.

### *2.3 Instruments and Procedures*

To comprehensively describe the instruments employed in this study, Table 1 summarizes the types, purposes, and illustrative examples of the key measurement and data collection tools used. These instruments collectively assess Computational Thinking (CT) components, monitor learning processes, and capture student perceptions, thereby providing multi-faceted evidence on the intervention's effectiveness. Sample items and rubric criteria included in the table exemplify how each instrument operationalizes the constructs under investigation and supports the reliability and validity of the data collected.

**Table 1.** Instruments Used in the Study

| Instrument Category | Purpose | Example/Key Features |
|---|---|---|
| CT Assessment (adapted from Shute et al., 2021; Weintrop et al., 2021) | Measures 5 CT components: abstraction, decomposition, algorithmic design, evaluation, generalization | Sample item: "Design an algorithm, using vector notation, for a drone to navigate between three GPS coordinates." Scored by correctness of decomposition, mathematical reasoning, and algorithm formulation. |
| Observation Rubric | Records engagement, collaboration, problem solving | Indicator: "Student interprets graphical solutions using GeoGebra. Level: 1 = rarely, 2 = sometimes, 3 = consistently." |
| LMS Analytics | Tracks digital interaction, time-on-task, task completion | Data: frequency of logins, assignment submissions, module completions. |
| Perception Survey | Captures student perspectives on model's usefulness and usability | Likert-scale (1–5): "The integration of APOS/RME made math topics more understandable." |

Sample CT assessment item:

"Given the task of programming a robot to move from Point A to Point C through Point B using vector operations, describe the necessary sequence of operations and justify your algorithm selection."

Sample rubric excerpt (Observation):

"Collaboration: Student communicates mathematical ideas with peers and justifies solution steps (1 = rarely, 3 = consistently)."

Instrument validity and reliability were confirmed by expert review (3 math education specialists, 2 digital learning experts). Inter-rater reliability for CT scoring was established via Cohen's kappa ($\kappa$ = 0.82). Instruments were pilot-tested for clarity and alignment.

### *2.4 Research Procedures*

The DBR cycle included:

a. Preliminary: Needs analysis, baseline CT assessment
b. Design: Model construction, instrument validation
c. Implementation: Six-week intervention across assigned class sections
d. Evaluation: Post-test, expert validation, iterative model refinement

The research procedure followed the Design-Based Research (DBR) cycle:

a. Preliminary phase: Baseline CT assessment and needs analysis.
b. Design phase: Model development integrating APOS, RME, and digital tools.
c. Implementation phase: Delivery of learning interventions across class sections over six weeks.

d. Evaluation phase: Post-intervention CT assessment, expert validation, and iterative refinements.

This systematic approach ensured both pedagogical rigor and empirical validity in evaluating the model's effectiveness.

### *2.5 Data Analysis*

Quantitative data (pre/post CT scores) were analyzed using descriptive statistics (means, SD, N-gain), paired t-tests, ANCOVA (baseline as covariate), and Cohen's d for effect size (Field, 2020). Qualitative data (observation rubrics, analytics, surveys) underwent thematic coding (Braun & Clarke, 2021), with triangulation among data sources (expert feedback, classroom observation, student surveys) to enrich findings. Integration of findings during reflection enabled iterative model improvements and ensured methodological rigour.

## 3. FINDINGS AND DISCUSSION

### *3.1 Gains in Computational Thinking (CT)*

Compared with the comparison class, the APOS–RME–integrated digital model yielded meaningful gains on all five CT components (Abstraction, Decomposition, Algorithmic Design, Evaluation, and Generalization) measured by pre–post CT assessments. Descriptively, normalized gains (N-gain) were in the moderate–high range for the experimental class (with the largest improvements typically on Decomposition and Algorithmic Design), while the comparison class showed low gains overall. ANCOVA (pretest as covariate) indicated significant posttest differences favoring the experimental group (see *Table 1*), and effect sizes (Cohen's *d*) were in the medium to large range for most CT components. These results align with the baseline profile obtained in the preliminary study where average scores were 2.4 (Abstraction), 2.7 (Decomposition), 2.5 (Algorithmic Thinking), 2.3 (Evaluation), and 2.6 (Generalization) on a 1–4.

**Table 2.** ANCOVA results for CT components

| CT Component | F(1, 70) | p-value | Partial $\eta^2$ |
|---|---|---|---|
| Abstraction | 15.62 | < .001 | 0.182 |
| Decomposition | 18.45 | < .001 | 0.209 |
| Algorithmic Thinking | 16.73 | < .001 | 0.193 |
| Evaluation | 12.58 | 0.001 | 0.158 |
| Generalization | 14.37 | < .001 | 0.171 |

*Note. Posttest as dependent variable; pretest as covariate; $\alpha = .05$.*

As shown in Table 2, ANCOVA results revealed statistically significant differences across all CT components between the experimental and comparison groups after controlling for pretest scores, with *F*-values ranging from 12.58 to 18.45 ($p < .001$). The largest effect was observed in Decomposition ($F = 18.45$, $p < .001$, partial $\eta^2 = 0.209$), indicating that the integrated APOS–RME digital model strongly influenced students' ability to break down complex vector problems into simpler subproblems. Pedagogically, the gains are consistent with the model's logic: (1) APOS sequences learning from Action→Process→Object→Schema, scaffolding the transition from procedures to concepts; (2) RME supplies realistic tasks (e.g., vector navigation, GPS mapping) that make abstractions meaningful; and (3) digital tools (LMS, GeoGebra, MATLAB) enable iterative exploration, immediate feedback, and structured practice. Together, these mechanisms create repeated opportunities for learners to externalize reasoning, test algorithms, and generalize across tasks—key processes that drive CT improvement.

The observed CT gains echo recent syntheses that emphasize technology-enhanced, theory-informed instruction for developing CT in higher education (Lu et al., 2023; Tariq et al., 2024). Studies that embed CT in mathematics report stronger outcomes when learners handle authentic problems with opportunities for algorithm design and representation shifts—features central to this model (Shute et al., 2021; Weintrop et al., 2021). Our findings also converge with reports that structured scaffolding accelerates progress from procedural manipulation to conceptual control (Tuktamyshov & Gorskaya, 2023) and that digital environments amplify feedback cycles and metacognition (Van den Heuvel-Panhuizen & Drijvers, 2022). Divergences with prior work mainly concern effect magnitude across CT facets: while some studies find the largest shifts on Abstraction, our cohort often improved most on Decomposition/Algorithmic Design—plausibly because tasks and analytics were tuned to breaking down vector problems and building stepwise procedures. Two limitations temper interpretation: first, intact-class assignment (cluster level) raises the possibility of section effects; second, exposure to tools (e.g., GeoGebra/MATLAB) varied at baseline, which, despite covariate control, may have introduced residual bias. Future work should randomize at the section level across multiple institutions and include delayed-post measures to test the durability of CT gains.

### *3.2 Conceptual Understanding in Vectors and Linear Transformations*

Performance on concept-focused items (vector operations, dot/cross products, linear combination, and transformation interpretation) improved significantly for the experimental group. Error analyses showed marked reductions in procedure–concept slippage (e.g., applying component-wise operations incorrectly), sign/confusion errors in vector addition/subtraction, and misinterpretations of orthogonality/parallelism when using dot products. Transfer items—mapping a real-world scenario (e.g., drone displacement with wind) to a formal vector model—also improved, suggesting better horizontal mathematization (moving from context to model) and vertical mathematization (moving toward abstraction). In-class artifacts revealed that students increasingly annotated transformations as functions on vectors and reasoned about images/pre-images under a matrix, indicating growth from APOS Action/Process to Object/Schema stages. Visual-interactive sequences in GeoGebra (dynamic arrows, sliders for components) and LMS-guided prompts likely supported conceptual anchoring and self-explanation, while MATLAB tasks encouraged students to express reasoning as computable procedures, tightening links between conceptual and algorithmic understanding.

**Table 3.** Effect sizes (Cohen's d) and N-gain by CT component

| CT Component | Cohen's d | N-gain (%) |
|---|---|---|
| Abstraction | 0.72 | 54.3 |
| Decomposition | 0.81 | 60.5 |
| Algorithmic Thinking | 0.76 | 58.2 |
| Evaluation | 0.64 | 50.1 |
| Generalization | 0.69 | 52.7 |

*Note. Interpretation of Cohen's d: small = 0.2, medium = 0.5, large = 0.8 (Cohen, 1988).*

Table 3 complements these findings by presenting effect sizes and normalized gains (N-gain) for each CT component. The greatest improvement occurred in Decomposition (Cohen's d = 0.81, N-gain = 60.5%), followed by Algorithmic Thinking (d = 0.76, N-gain = 58.2%). All effect sizes were in the medium-to-large range, confirming that the intervention not only produced statistically significant gains but also yielded practically meaningful improvements in students' CT skills.

The shift in vector/linear-transformation understanding aligns with research showing that interactive visualization and structured progression enhance conceptual grasp in abstract mathematics (Van den Heuvel-Panhuizen & Drijvers, 2022; Fukui et al., 2023). Prior APOS-oriented interventions report gains when instruction explicitly orchestrates transitions from manipulative actions to process encapsulation and object formation (Tuktamyshov & Gorskaya, 2023). Likewise, RME studies emphasize that modeling from realistic contexts sustains meaning as learners formalize ideas (Van Zanten & Van Den Heuvel-Panhuizen, 2021). Our results are consistent with that trajectory: students leveraged context to stabilize meaning, then used tools to climb the ladder of abstraction. Differences with some reports (that find limited transfer to novel tasks) may reflect our deliberate spiraling of contexts and prompted generalization activities embedded in the LMS, which repeatedly cued students to compare representations (diagram, component form, matrix form). Limitations include reliance on course-embedded assessments (risking alignment with taught content) and potential practice effects due to iterative tasks. To mitigate these, we included novel transfer items and rubric-based scoring with inter-rater reliability checks. Future studies should incorporate external concept inventories and cross-topic transfer (e.g., eigen-interpretations) to test the breadth of conceptual change.

### *3.3 Engagement and Learning Process Analytics*

LMS analytics indicated higher engagement intensity in the experimental class—greater session counts, time-on-task, and completion rates for exploratory activities. Heatmaps of resource usage showed repeated visits to concept-building pages and interactive applets, aligning with the model's APOS staging (Action practice → Process reflection → Object consolidation). Discussion-board traces revealed more algorithm sketches and peer critiques in the experimental class, while observation rubrics recorded increases in collaboration and justification quality. These process signals correspond with CT assessment gains: learners who spent more time on decomposition prompts and algorithmic planning checklists tended to show larger posttest improvements in Algorithmic Design and Evaluation. The proceduralization of reasoning in MATLAB notebooks—where students codified steps and tested edge cases—appears to have strengthened metacognitive monitoring (debugging and evaluation). Overall, analytics portray a productive struggle pattern: learners iterated between visual exploration (GeoGebra), procedural expression (MATLAB), and reflective checkpoints (LMS quizzes), a sequence that plausibly deepened CT.

Observed engagement patterns echo evidence that well-designed digital ecosystems can move technology use beyond delivery to cognitive orchestration—linking resources, prompts, and feedback to higher-order outcomes (Van den Heuvel-Panhuizen & Drijvers, 2022; Fukui et al., 2023). Reviews in CT education emphasize the role of structured tasks, scaffolded planning, and iterative feedback in building algorithmic thinking and decomposition (Lu et al., 2023; Tariq et al., 2024). Our analytics–outcomes correlations converge with this literature, suggesting that time in exploratory/problem-structuring activities is a meaningful predictor of CT gains. Where we diverge from some reports that show plateauing engagement mid-semester, our weekly reflection prompts and low-stakes checkpoints may have sustained participation. Limitations include possible self-selection effects (motivated students engage more) and the interpretive nature of learning-analytics proxies. We mitigated these by (a) triangulating analytics with classroom observations and artifact coding and (b) relating analytics to objective assessment gains rather than treating them as outcomes. Future work can apply causal learning analytics designs or A/B tests on prompt timing to identify which orchestration moves most powerfully drive CT development.

### *3.4 Perceptions, Feasibility, and Expert Validation*

Student surveys reported high perceived usefulness and ease of use for the integrated model, with particular value placed on authentic real-world tasks and visual-interactive components. Open-ended responses credited scenario realism (e.g., navigation with wind) for deepening conceptual understanding of vector operations, while APOS-style sequencing clarified each procedural step. Observation data confirmed that the intervention was feasible within standard class periods; instructors emphasized benefits such as reusable LMS templates, GeoGebra applets, and MATLAB notebooks, which reduced preparation time after initial setup.

To facilitate scale-up, the onboarding process can be formalized via structured digital orientation modules, step-by-step demonstration videos, and sequenced hands-on sessions at the start of implementation cycles. Such approaches have been shown to mitigate early adoption challenges and can be institutionalized as part of phased digital tool integration (Saadah & Indrawatiningsih, 2024; Clark-Wilson, 2024). In this study, onboarding supports—including paired peer guidance and offline guides—helped bridge initial disparities in digital skills and access, rapidly reducing early friction and enabling more equitable participation (Latysheva et al., 2021; Lu et al., 2023). Expert validators in mathematics education and educational technology rated the content validity and instructional coherence of the model as high, noting clear conceptual alignment from activities to CT indicators.

Despite overall positive perceptions and feasibility, several recurring challenges—device access, heterogeneous digital skills, and initial resistance—were reported by students and faculty, echoing recent large-scale reviews (Johnson et al., 2020; Abejuela et al., 2022). Table 4 presents a summary of the main implementation challenges and the strategies used to address them, drawn from both the present study and contemporary literature (2021–2025):

**Table 4.** Implementation Challenges and Mitigation Strategies

| Challenge | Mitigation Strategy |
|---|---|
| Limited device access | Offline guides (PDF, screenshots); peer group tasks |
| Digital skills disparity | Modular tool onboarding; peer mentoring |
| Early adoption resistance | Orientation sessions; instructor scaffolding |
| Time constraints | Reusable LMS shells and applet libraries |
| Instructor workload | Editable resource packages; collaborative planning |

The provision of a reusable implementation package—comprising LMS shells, applet libraries, and concise digital training guides—is a notable strength for facilitating iterative, scalable deployment in future cycles (Aquino, 2024; Zana et al., 2024). However, limitations remain, such as single-institution sampling and short intervention periods, restricting generalizability and long-term impact assessment. Future research should address sustainability across curricula, departmental scaling, and equity of access, including tailored supports for students with limited devices or prior digital experience.

In sum, with formalized onboarding and reusable digital resources, the model is both instructionally promising and operationally viable for broader mathematics education settings. Continued refinement is necessary to optimize scalability and equitable implementation (Lu et al., 2023; Clark-Wilson, 2024).

## 4. CONCLUSION

This study successfully designed, implemented, and evaluated a strategic digital learning model integrating APOS theory, Realistic Mathematics Education (RME), and digital tools to enhance university students' Computational Thinking (CT). The model demonstrated pedagogical coherence and practical feasibility, leading to significant improvements across CT components—abstraction, decomposition, algorithmic design, evaluation, and generalization—as well as deeper conceptual understanding of abstract mathematics topics such as vectors and linear transformations. The design-based research approach allowed iterative refinement, ensuring alignment of activities and assessments with CT indicators and the APOS–RME learning progression from contextual problems to formal mathematical representations. Theoretically, the findings highlight the importance of integrating cognitive and contextual theories within a digital ecosystem to make CT a teachable, assessable competency rather than an incidental outcome of programming alone. Practically, the coordinated use of LMS prompts, GeoGebra visualizations, and MATLAB notebooks effectively mediated students' exploration, formalization, and evaluation processes, while reusable digital resources facilitated scalable and consistent implementation. Despite positive outcomes, challenges such as onboarding disparities, device access, and instructor preparedness were addressed through structured tool orientation, peer support, and offline materials, suggesting that formalized onboarding processes and implementation packages are key for sustainable scale-up. Future research should expand testing across diverse courses and institutions, strengthen assessment through advanced psychometric methods, develop comprehensive training programs for instructors and students, and prioritize equity by providing low-bandwidth and offline alternatives. Maintaining continuous design-based cycles informed by learning analytics and expert feedback will support ongoing model optimization. Institutional support and data-driven monitoring remain essential to overcome infrastructural and pedagogical barriers, ensuring the model's long-term viability and contribution to cultivating 21st-century computational skills in higher education mathematics.

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